# SHORTER CONTRIBUTIONS 

# THE DISTRIBUTION OF RAINDROPS WITH SIZE 

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Measurements oi raindrop records on dyed filter papers were made for correlation with radar echoes (Marshall, Langille, and Palmer, 1947) These measurements have been analyzed to give the distribution at drops with size (fig, 1). The distributions are in fair agreement with those of Laws and Parsons (1943).


FIG. 1. Distribution of number versus diameter for raindrops recorded at Ottawa, summer 1946. Curve A is for rate of rainfall $1.0 \mathrm{~mm} \mathrm{hr}^{-1}$, curves $\mathrm{B}, \mathrm{C}, \mathrm{D}$, for $2.8,6.3,23.0 \mathrm{~mm} \mathrm{hr}^{-1}$. $N_{D} \delta D$ is the number of drops per cubic meter, of diameter between D and $\mathrm{D}+\delta D \mathrm{~mm}$. Multiplication by $10^{-6}$ will convert $N_{D}$ to the units of equation (2).
Except at small diameters, both sets of experimental observations can be fitted (fig, 2) by a general relation,

$$
\begin{equation*}
N_{D}=N_{0} e^{-\Lambda D} \tag{1}
\end{equation*}
$$

where $D$ is the diameter, $N_{D} \delta D$ is the number of drops of diameter between $D$ and $D+\delta D$ in unit volume of space, and $N_{0}$ is the value of $N_{D}$ for $D=0$.
It is found that

$$
\begin{equation*}
N_{0}=0.08 \mathrm{~cm}^{-1} \tag{2}
\end{equation*}
$$

for any intensity of rainfall, and that

$$
\begin{equation*}
\Lambda=41 R^{-0.21} \mathrm{~cm}^{-1} \tag{1}
\end{equation*}
$$

where $R$ is the rate of rainfall in $\mathrm{mm} \mathrm{hr}^{-1}$.
For diameters less than about 1.5 mm , both sets of observations fall short of the value for $N_{D}$ given by equation (1), and they disagree slightly with each other. Laws and Parsons' observations are better in
this region, and tend toward a common value of $N_{0}$ for all rates of rainfall.
The mass of rain water $M$ per unit volume of space, and the sum $Z$ of sixth powers of drop diameters in unit volume (a radar quantity), can be calculated as functions of $\Lambda$ from equation (1), and so correlated with the rate of rainfall $R$ by equation (3). It is of interest to compare these correlations with those obtained when $M, Z$, and $R$ are determined more directly from the experimental records (table 1). The deficit of

TABLE 1. $M=1 / 6 \pi \Sigma N_{D} D^{3} \delta D$ and $Z=N_{D} D^{6} \delta D$ as functions of the rate of rainfall $R$

| Reference | M <br> $\mathrm{mgm} \mathrm{m}^{-3}$ | Z <br> $\mathrm{mm}^{6} \mathrm{~m}^{-3}$ |
| :--- | :---: | :---: |
| Marshall, Langille and Palmer (1947) | $80 R^{0.85}$ | $190 R^{1.72}$ |
| Revision of the above | $72 R^{0.88}$ | $220 R^{1.60}$ |
| Z/R correlation by Wexler (1947) | $68 R^{0.88}$ | $320 R^{1.44}$ |
| (data of Laws and Parsons (1943) | $89 R^{0.84}$ | $296 R^{1.47}$ |

small drops in the observations, as compared with equation (1), should make the observed value of $M$, and to a lesser extent that of $Z$, smaller than those derived from the equations.


FIG. 2. Distribution function (solid straight lines) compared with results of Laws and Parsons (broken lines) and Ottawa observations (dotted lines).

[^0]The exponential distribution of equation (1) is the type that would obtain if growing drops were.in continual danger of disintegrating, the likelihood of disintegration being proportional to the increment in diameter or in distance of fall through cloud. Such behavior might be explained by the random accumulation by each drop of electrical charge as more and more randomly charged cloud drops or smaller raindrops are acquired by coalescence, and the resultant disintegration of overcharged drops. Relevant calculations and experiments on coalescence are in progress.

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